Week 7 PSO Questions

TAs

Mukul Agarwal Aaryan Wadhwani

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Coffee Before Class. A campus coffee stand earns \$2 per regular coffee and \$3 per fancy coffee. Before class, it can make at most 4 drinks in total, of which at most 2 can be fancy coffees.

Let x be the amount of regular coffee and y the amount of fancy coffee. Allow fractional amounts. The linear program is

$$\begin{aligned} \text{maximize} \quad & 2x + 3y \\ \text{subject to} \quad & x + y \le 4, \\ & y \le 2, \\ & x \ge 0, \quad y \ge 0. \end{aligned}$$

1. Shade the feasible region in the (x, y)-plane. Its vertices are $(0, 0)$, $(4, 0)$, $(2, 2)$, and $(0, 2)$.
2. Compute the profit at each vertex. Which point gives the largest profit, and how many of each drink should the stand make?

Hint. A linear objective on this feasible region attains its maximum at a vertex.

Minimizing Distance. Given $n \geq 1$ distinct integers $x_1, ..., x_n$, we want to find an integer y that is close to the input values. For each part, give an $O(n)$ time algorithm and prove its correctness.

1. Find an integer y that minimizes the sum of squared distances:
$$\sum_{i=1}^{n} (y - x_i)^2.$$
2. What if we were instead minimizing the following function?
$$\max_i |y - x_i|$$
3. What if we were instead minimizing the following function?
$$\sum_{i=1}^{n} (y - x_i)^4$$