Week 5 PSO Questions

TAs

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Definition **Subsequence of an Array**

A **subsequence** of an array $A[1..n]$ is a sequence $A[i_1], ..., A[i_k]$ such that

$$1 \leq i_1 < i_2 < ... < i_k \leq n.$$

Longest Increasing Subsequence. Let $A[1..n]$ be an array. We say that a subsequence of A (indexed by $i_1, ..., i_k$) is increasing if and only if

$$A[i_j] \leq A[i_{j+1}] \quad \text{for every } j \in \{1, ..., k-1\}.$$

Design and analyze an algorithm to compute the length of the *longest* increasing subsequence of A.

Longest American Increasing Subsequence.[1] Suppose each entry in A is also colored red, white, or blue. We say that a sequence is American if the colors alternate red, white, blue, red, white, blue, … The first number in the sequence can be any color. Compute the length of the longest increasing American subsequence of A. (You can assume that you can look up the color of $A[i]$, for any $i \in [n]$, in constant time.)

Just For Fun.[2] Can you find an algorithm *faster* than the basic dynamic program we discussed in PSO?

[1] This extension of the LIS problem is due to Kent Quanrud.

[2] *Hint.* Think of Binary Search.